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Generalized Poynting Effects in Predeformed Prismatic Bars

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This presentation is based on the paper [1]. Already Poynting observed that an elastic prismatic bar when twisted by torques applied at the end faces also elongates, and the elongation is proportional to the square of the twist. This observation was framed in the theory of Continuous bodies by Rivlin who used the second-order elasticity theory to compute the elongation as a function of the angular twist and showed that the elongation was proportional to second-order elasticities. The results by Rivlin has been extended to general nonlinear elastic solids by Green and Shield. Subsequently the Signorini's method for studying the deformation of non-linear continua was used to deal with Poynting type effects: Green and Adkins noted that when the displacements and infinitesimal rotations of the centroid of one end-face vanish, then the compatibility condition for the loads in the Signorini's expansion method is automatically satisfied. As Signorini's method reduces the solution of a nonlinear elastic problem to that of a series of linear elastic problems with body forces and surface tractions determined by the solution of the previous linear elastic problem it is suitable to be used in conjunction with the Saint-Venant method for studying the linear elastic deformations of bodies of prismatic form.

Those interested in more details about the literature in the subject can refer to Truesdell and Noll who have reviewed the historical background and the relevant literature on the Poynting effect and the Signorini expansion method. They have pointed out that Signorini's method delivers only those solutions that are in the neighborhood of solutions of the linear elastic problem with the same loads as for the nonlinear problem.

In this paper we investigate the problem when the first term in Signorini's expansion corresponds to an infinitesimal bending and stretching of the bar. The solution of the problem yields generalized Poynting effects such as second order extension of the bar in the absence of external resultant normal tractions at the end faces. This second- order extension is found to depend upon the first and second-order elasticities, initial bending, and other properties of the crosssection. Further such effects are deduced for a pretwisted bar.

The analysis of the problem is simplified by decomposing the displacement field and other tensorial quantities into components along the axis of the bar and those in the cross-section.

We have found a second-order solution of the Saint-Venant problem for a straight, prismatic, homogeneous body made of a second-order elastic material.

The bar is initially stretched and bent by an infinitesimal amount from the unstressed reference configuration, and then deformed by tractions applied at the end faces. The displacements and infinitesimal rotations at the centroid of one end face are taken to be zero.

The problem is analyzed by the Signorini expansion method, and its solution reduced to that of solving two linear plane elliptic problems. The resultant second-order forces and moments at an end face are given by equations are explicitly calculated. Also a new Poisson's effect proportional to the square of the initial infinitesimal deformation is found .

The Poisson effect in a cross-section varies as the square of its distance from the 'fixed' end, and the in-plane displacement of a point in the bending direction is more than that in the orthogonal direction. Unless the direction of prebending is along a principal axis of a cross-section, the cross-section twists in the absence of external torques at the end faces.

This twist is proportional to the square of the distance of the cross-section from the 'fixed' end, and is determined by Poisson's ratio, the bending vector and properties of the cross-section, but is independent of the second-order elasticities. Moreover we prove that

I) in the absence of a second-order resultant normal force, there can be a second-order extension of the bar which depends upon the first and second order elasticities of the material, the bending vector and geometric properties of the cross-section

II) a second-order bending effect may occur in the absence of corresponding bending moments at the end faces.

III) for a pretwisted bar, there is a second-order bending deformation for vanishing second-order bending moments at the end faces.

The importance of the semi-analytical methods which we present here cannot be underestimated also in the époque of intensive use of numerical methods. Indeed optimization problems and parametric analyses are still greatly facilitated by an a priori knowledge of families of solutions. Moreover the solutions found with semi-analytical methods can be fruitfully used as benchmarks for newly conceived numerical techniques.

References

[1] F. dell'Isola, G. Ruta, and R. Batra. Generalized poynting effects in predeformed prismatic bars. *Journal of Elasticity*,50: 181-196, 1998.

[2] A.E. Green and J.E. Adkins. Large Elastic Deformations and Nonlinear Continuum Mechanics. Claredon Press, Oxford, 1960.

[3] A. Signorini. Sulle deformazioni termoelastiche finite. *Proc. 3rd Int. Congr. Appl. Mechs,* 2: 80–89, 1930.